Macroscopically frustrated Ising model

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A disordered spin-glass model in which both static and dynamical properties depend on macroscopic magnetizations is presented. These magnetizations interact via random couplings and, therefore, the typical quenched realization of the system exhibits a macroscopic frustration. The model is solved by using a revisited replica approach, and the broken symmetry solution turns out to coincide with the symmetric solution. Some dynamical aspects of the model are also discussed, showing how it could be a useful tool for describing some properties of real systems such as, for example, natural ecosystems or human social systems.

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I. INTRODUCTION

Macroscopic frustration can be found in different domains, from interpersonal relationships to granular matter or natural ecosystems. All these systems are characterized by frustrated components with a thermodynamically macroscopic size. In other words, in all these systems, there are components whose size is comparable with that of the whole system and that cause the action of opposite forces. The classical example is the case of a man A who desires to spend some time with a dear friend B, who, unfortunately, wants to bring his wife C, who is really detested by A.

Dozens of examples can be found in nature. Consider the antlers of a deer. It is known that they represents a frustrated phenotype. In fact, sexual selection tends to prefer their growth in order to increase the chance of reproduction, but antlers are an obstacle in some situations, such as with a predator pursuit in a forest, and, therefore, natural-selection pressure is for their reduction.

From a stricter physical point of view, systems that exhibit frustration are very common (see [1] for a general view). For a disordered spin system, Toulouse [2] has introduced the definition of frustration for an elementary plaquette of bonds, consisting in the product of the corresponding couplings. Nevertheless, systems in which frustration appears on macroscopic scales are less ordinary and have not yet been investigated, as far as we know.

In this paper, we present a spin-glass model in which spins are organized in macroscopic sets, with the corresponding macroscopic magnetizations interacting via random couplings. For a typical random realization of the couplings, the system is an ensemble of interacting frustrated macroscopic entities and, therefore, it could be a natural candidate for the mathematical modeling of phenomena in which macroscopic frustration plays a central role.

Let us briefly sum up the contents of the paper.

In Sec. II, the model is introduced. The model becomes self-averaging when the number of components is large, nevertheless some considerations about its finite-size version are also noted.

In Sec. III, we look for a solution of the model using a revisited version of the replica trick. This revised version could be applied in a more general context to a large class of models, as will be explained.

In Sec. IV and Sec. V, respectively, the replica symmetry solution and the broken symmetry solution in the manner of Parisi are derived in detail. The two solutions turn out to coincide, thus negating the benefits that the Parisi ansatz has in other spin-glass models.

In Sec. VI, the symmetric solutions is studied in detail from a numerical point of view showing that, at variance with the Sherrrington-Kirkpatrick (SK) model, it keeps its physical meaning even at very low temperature.

In Sec. VII, some final remarks are made. In particular, some dynamical aspects are illustrated. Dynamics could be a profitable argument for future investigations, especially for its possible applications to ecosystems and natural-selection modeling.

II. THE MODEL

Let us consider a Hamiltonian where N spins are divided in L sets, each set consisting of exactly $M=N/L$ spins. Each spin interacts with all other spins, but the coupling does not depend on the sites of the spins, but only on the sets of the spins involved. In other words, two spins of different sets interact via a coupling that depends only on the coordinates of the two sets of membership. Then, we can speak of coupling between sets rather than between spins. We also assume that spins of the same set do not interact.

This Hamiltonian can be written as

$$H_{M,L}(J,\sigma)=-\frac{1}{M\sqrt{L}}\sum_{k>l} J_{k,l}\sigma_k\sigma_l, \tag{1}$$

where J is an $N\times N$ symmetric matrix, consisting of L^2 blocks of M^2 entries each, M being the linear size of a block. All the M^2 entries of a given block take the same value and, in particular, the diagonal blocks consist of null entries. The free energy of the system is

$$f_{M,L}(J)=-\frac{1}{\beta ML}\ln\sum_{\{\sigma\}} \exp[-\beta H_{M,L}(J,\sigma)], \tag{2}$$

where the sum is intended over all the spin configurations.

The thermodynamic limit $N\to\infty$ can be obtained in two different ways since N is the product of two variables ($N=LM$). In fact, the limit $L\to\infty$ would mean considering a system whose properties and characteristics are the same of those of the SK model [3]. On the contrary, the limit $M\to\infty$ leads to a mean-field model with a macroscopic frustration. The self-average properties are obtained by also performing the limit $L\to\infty$ after the limit $M\to\infty$. Nevertheless, non-self-averaging macroscopic frustration is also exhibited for finite L, as we will show later with an example.

We thus perform the limit $M\to\infty$, keeping L finite. After some algebra, the free energy reads

$$f_L(J)=-\frac{1}{\beta L}\max_{\mathbf{m}}\Gamma(J,\mathbf{m}),$$

where $\mathbf{m}=(m_1,\ldots,m_L)$, having defined the ith set magnetization m_i as

$$m_i=\lim_{M\to\infty}\frac{1}{M}\sum_{k\in i\text{th set}}\sigma_k,$$

and where

$$\Gamma(J,\mathbf{m})=\frac{\beta}{\sqrt{L}}\sum_{i>j}J_{i,j}m_im_j+\sum_i\Phi(m_i). \tag{3}$$

The indices i and j run over the spin sets, and J is now a symmetric $L\times L$ matrix, obtained from the matrix in Eq. (1) substituting each block with a single entry, $J_{i,j}$ being the value of the coupling connecting a spin of set i with a spin of set j, with $J_{i,i}=0\,\forall i$. Furthermore, $\Phi(m_i)$ represents the entropic term of spin set i,

$$\Phi(m_i)=-\frac{1+m_i}{2}\ln\frac{1+m_i}{2}-\frac{1-m_i}{2}\ln\frac{1-m_i}{2}.$$

Let us suppose that the nondiagonal elements of J are independent, identically distributed random quenched variables. For the sake of simplicity, we restrict ourselves to considering normal Gaussian variables with vanishing average and unitary variance. Our aim is to compute the quenched free energy

$$f=\lim_{L\to\infty}f_L(J)=-\lim_{L\to\infty}\frac{1}{\beta L}\overline{\max_{\mathbf{m}}\Gamma(J,\mathbf{m})}, \tag{4}$$

where the last equality is due to the self-averaging property of the free energy, which holds in the large-L limit. The max in Eq. (4) is reached for $\mathbf{m}^*=(m_1^*,\ldots,m_L^*)$, which obeys the following L self-consistent equations:

$$m_i^*=\tanh\left[\frac{\beta}{\sqrt{L}}\sum_j J_{i,j}m_j^*\right],\quad 1\leq i\leq L. \tag{5}$$

We consider the large-L limit, because we have in mind a system with many macroscopic frustrated components, nevertheless the glassy characteristics (except self-averaging) can also be found for finite L. Consider, for instance, $L=3$ with the product of the three couplings with a negative sign. At low temperature (below transition, not vanishing), the system is degenerated since it has six different pure states with the same free energy and with nontrivial and not all equal values of the three magnetizations involved.

When L increases, frustration increases, as does the number of pure states corresponding to the same free energy. We hope to find in this way an interesting spin-glass model with new peculiarities.

III. REPLICA TRICK REVISITED

In order to perform the limit $L\to\infty$, we need to compute $\overline{\max_{\mathbf{m}}\Gamma(J,\mathbf{m})}$. We will accomplish this task by means of the replica trick with a slight but crucial variant. Let us stress from the beginning that this way of applying the replica trick is not restricted to our model, but it is more general and, in principle, could be of some help in solving many other models with macroscopic variables. In fact, what we propose here is a useful technique for computing quantities of the type $\overline{\max_{\mathbf{m}}\Gamma(J,\mathbf{m})}$, i.e., an average whose argument is a maximum over an expression that depends on random variables (J) and on variables to be maximized ($\mathbf{m}$).

It is easy to check that

$$\overline{\max_{\mathbf{m}}\Gamma(J,\mathbf{m})}=\lim_{\mu\to\infty}\lim_{n\to0}\frac{1}{\mu n}\ln\overline{\left[\int d\mathbf{m}\exp(\mu\Gamma(J,\mathbf{m}))\right]^n}, \tag{6}$$

where $d\mathbf{m}=\Pi_i dm_i$. In fact, after having performed the limit $n\to0$ as in ordinary replica trick on the right-hand side of Eq. (6), the saddle-point method allows us to compute the limit $\mu\to\infty$, giving equality (6). The variable μ here is only an auxiliary one.

Making explicit the n replicas, the average on the right-hand side of Eq. (6) can be written as

$$\overline{\left[\int d\mathbf{m}\exp(\mu\Gamma(J,\mathbf{m}))\right]^n}=\int\prod_\alpha d\mathbf{m}^\alpha\exp G_n(\mu,\mathbf{m}^1,\ldots,\mathbf{m}^n),$$

having defined

$$G_n(\mu,\mathbf{m}^1,\ldots,\mathbf{m}^n)\equiv\ln\overline{\exp\sum_\alpha\mu\Gamma(J,\mathbf{m}^\alpha)},$$

where the index α runs over the n replicas. Finally, this leads to the following expression for the free energy f:

$$f=-\lim_{L\to\infty}\lim_{\mu\to\infty}\lim_{n\to0}\frac{1}{\beta L\mu n}\times\ln\int\prod_\alpha d\mathbf{m}^\alpha\exp G_n(\mu,\mathbf{m}^1,\ldots,\mathbf{m}^n). \tag{7}$$

In our case, keeping in mind Eq. (3), we can give an explicit expression for G_n. For the sake of simplicity, we do not write in the following the argument of G_n. After have taking the averages over the Gaussian J variables, and after some algebra, one has

$$G_n=\frac{\mu^2\beta^2}{4L}\sum_{\alpha,\alpha'}\left(\sum_i m_i^\alpha m_i^{\alpha'}\right)^2+\mu\sum_{i,\alpha}\Phi(m_i^\alpha),$$

where α and α' run over the replicas, and where terms not diverging with L have been neglected since they would disappear in the successive limit $L\rightarrow\infty$. By means of the parabolic maximum trick, the above expression can be rewritten as

$$G_n=\max_{\{q_{\alpha,\alpha'}\}}\left[\frac{\mu^2\beta^2}{2}\sum_{\alpha,\alpha'}\left(q_{\alpha,\alpha'}\sum_i m_i^\alpha m_i^{\alpha'}-\frac{L}{2}q_{\alpha,\alpha'}^2\right)+\mu\sum_{i,\alpha}\Phi(m_i^\alpha)\right],$$

where $\{q_{\alpha,\alpha'}\}$ is an $n\times n$ matrix, which represents from a physical point of view the overlap between replicas in spin-glass theory.

Now the integral in Eq. (7) can be fully factorized among the different spin sets, individuated by the index i. This fact allows us to perform the limit $L\rightarrow\infty$, which gives the final expression for the free energy in the replica context:

$$f=-\max_{\{q_{\alpha,\alpha'}\}}\lim_{\mu\rightarrow\infty}\lim_{n\rightarrow 0}\frac{1}{\beta\mu n}\ln\int\prod_\alpha dm^\alpha\exp\tilde{G}_n \tag{8}$$

with

$$\tilde{G}_n=\frac{\mu^2\beta^2}{2}\sum_{\alpha,\alpha'}\left(q_{\alpha,\alpha'}m^\alpha m^{\alpha'}-\frac{1}{2}q_{\alpha,\alpha'}^2\right)+\mu\sum_\alpha\Phi(m^\alpha),$$

where now $m^1,\ldots,m^n$ are n replicas of a scalar magnetization. Notice that interchange of the position between the $\max_{\{q_{\alpha,\alpha'}\}}$ and the integration is allowed since in the limit $L\rightarrow\infty$ this maximum corresponds to a saddle-point approximation of an integration with respect to the same variables $\{q_{\alpha,\alpha'}\}$.

IV. REPLICA SYMMETRIC SOLUTION

In order to find a solution, i.e., to compute the quenched free energy (8), we start by trying the usual symmetry unbroken strategy. Let us stress that the diagonal terms of matrix q are relevant for this model, at variance with the celebrated replica solution of the SK model [3]. Therefore, in spite of assuming that the diagonal terms vanish as in the symmetry-unbroken solution of SK, we assume

$$q_{\alpha,\alpha'}=q+\frac{x}{\beta\mu}\delta_{\alpha,\alpha'},$$

where $\delta_{\alpha,\alpha'}$ is the Kronecker delta. Notice that elements on the diagonal differ only for a quantity of the order of μ^{-1} from the other entries, otherwise one would have diverging terms in the limit $\mu\rightarrow\infty$. This fact implies that overlap turns out to be a constant only once the limit $\mu\rightarrow\infty$ has been performed. With this choice, one gets

$$\tilde{G}_n=\left[\frac{\mu^2\beta^2}{2}q\left(\sum_\alpha m^\alpha\right)^2+\frac{\mu\beta x}{2}\sum_\alpha[(m^\alpha)^2-q]\right]+\mu\sum_\alpha\Phi(m^\alpha),$$

where terms that vanish in the two limits $n\rightarrow 0$ and $\mu\rightarrow\infty$ have been neglected. By means of the standard Gaussian trick, we have

$$\exp\left[\frac{\mu^2\beta^2}{2}q\left(\sum_\alpha m^\alpha\right)^2\right]=\left\langle\exp\left(\mu\beta\omega\sqrt{q}\sum_\alpha m^\alpha\right)\right\rangle, \tag{9}$$

where the average $\langle\rangle$ is on an independent normal Gaussian variable ω. The above equality allows for writing

$$\exp\tilde{G}_n=\left\langle\prod_\alpha\exp\left(\mu\beta\omega\sqrt{q}m^\alpha+\frac{\mu\beta x}{2}[(m^\alpha)^2-q]+\mu\Phi(m^\alpha)\right)\right\rangle.$$

Notice that the argument inside the $\langle\rangle$ average in the preceding expression is fully factorized among the n replicas. For this reason, the integral in Eq. (8) becomes the nth power of a single integral, and therefore the limit $n\rightarrow 0$ can be performed:

$$f=-\max_{q,x}\lim_{\mu\rightarrow\infty}\frac{1}{\mu\beta}\left\langle\ln\int dm\exp\left[\mu\beta\omega\sqrt{q}m+\frac{\mu\beta x}{2}(m^2-q)+\mu\Phi(m)\right]\right\rangle.$$

Finally, the limit $\mu\rightarrow\infty$ can be performed by means of the saddle-point technique, obtaining

$$f=-\max_{q,x}\left\langle\max_m\left[\omega\sqrt{q}m+\frac{x}{2}(m^2-q)+\frac{\Phi(m)}{\beta}\right]\right\rangle. \tag{10}$$

Let us stress once again the important role played by the small symmetry breaking (nonvanishing x) introduced in the overlap. In fact, if we set $x=0$, choosing in this way a pure unbroken solution, the extremization with respect to q would be impossible, since the argument in Eq. (10) would diverge for $q\rightarrow\infty$. It also should be noticed that at least one of the maxima with respect to q and x could become a minimum after performing the limit $n\rightarrow 0$.

V. FAILURE OF BREAKING

Trying to apply the ordinary approach to spin-glass models, the following step consists in introducing an asymmetry in the overlap matrix. Assume now that

$$q_{\alpha,\alpha'}=q+\frac{x}{\beta\mu}\delta_{\alpha,\alpha'}+\frac{y}{\beta\mu}\gamma_{\alpha,\alpha'}.$$

Following Parisi parametrization [4–8], $\gamma_{\alpha,\alpha'}$ is a matrix whose entries vanish except in n/l quadratic blocks of l^2 elements along the diagonal, where all entries are equal to 1. Notice that we have made explicit once again a factor μ^{-1}, otherwise we would have divergent terms. In this case, the maximum has to be taken with respect to q, x, y, and l.

With this ansatz and neglecting terms vanishing in the successive limits $n\to 0$ and $\mu\to\infty$, $\tilde{G}_n$ turns out to be

$$\tilde{G}_n=\frac{\mu^2\beta^2}{2}q\left(\sum_{\alpha} m^{\alpha}\right)^2+\frac{\mu\beta x}{2}\sum_{\alpha}[(m^{\alpha})^2-q]+\frac{\mu\beta y}{2}\sum_{k}\left[\left(\sum_{\alpha\in k} m^{\alpha}\right)^2-ql^2\right]+\mu\sum_{\alpha}\Phi(m^{\alpha}),$$

where the index k runs over the n/l blocks on the diagonal of $\gamma_{\alpha,\alpha'}$ and the sum on $\alpha\in k$ goes on the l values of α corresponding to the kth block.

By means of the parabolic maximum trick, it is possible to write

$$\left[\frac{\mu\beta y}{2}\left(\sum_{\alpha\in k} m^{\alpha}\right)^2\right]=\max_{\rho_k}\left[\sqrt{\mu\beta y}\,\rho_k\sum_{\alpha\in k} m^{\alpha}-\frac{\rho_k^2}{2}\right].$$

In this way, repeating also the trick in Eq. (9), we have factorized $\tilde{G}_n$ with respect to the n/l blocks, and, therefore, the limit $n\to 0$ can be performed. One gets

$$f=-\max_{q,x,y,l}\lim_{\mu\to\infty}\frac{1}{\mu\beta l}\left\langle \ln\int\prod_{\alpha} dm^{\alpha}\max_{\rho}\hat{G}_n\right\rangle, \qquad (11)$$

with

$$\hat{G}_n=\sum_{\alpha}\left[\mu\beta\omega\sqrt{q}m^{\alpha}+\frac{\mu\beta x}{2}[(m^{\alpha})^2-q]+\sqrt{\mu\beta y}\rho m^{\alpha}-\frac{\rho^2}{2l}-\frac{\mu\beta y}{2}ql+\mu\Phi(m^{\alpha})\right],$$

where now the index α runs over only a single block, which corresponds to the scalar variable ρ, and where $\langle\rangle$ means the average over the normal Gaussian ω.

The $\max_{\rho}$ in Eq. (11) can be put outside the integration. This change is allowed and can be understood by the same argument used after Eq. (8). As a consequence, the integral in the preceding expression is factorized among the l replicas of a block, and reduces to a single integral because of the factor l in the denominator. Moreover, this integral can be computed by means of the saddle-point method in the limit $\mu\to\infty$, obtaining

$$f=-\max_{q,x,y,l}\lim_{\mu\to\infty}\frac{1}{\mu\beta}\left\langle\max_{\rho,m}\left[\mu\beta\omega\sqrt{q}m+\frac{\mu\beta x}{2}(m^2-q)+\sqrt{\mu\beta y}\rho m-\frac{\rho^2}{2l}-\frac{\mu\beta y}{2}ql+\mu\Phi(m)\right]\right\rangle.$$

The maximum with respect to ρ can be computed, and then performing the limit $\mu\to\infty$, one finally has

$$f=-\max_{q,x,y,l}\left\langle\max_{m}\left[\omega\sqrt{q}m+\frac{x+yl}{2}(m^2-q)+\mu\Phi(m)\right]\right\rangle.$$

Unfortunately, this final result is exactly the same as that of the unbroken case (10), the only difference being that the variable x is substituted by $x+yl$, which is irrelevant when the maximum is taken.

This result could imply that the model simply has a constant overlap that depends only on the temperature; otherwise, one should admit that the Parisi ansatz for replica symmetry breaking is inappropriate in this context.

VI. UNDERSTANDING THE REPLICA SYMMETRIC SOLUTION

The unfortunate result of the replica broken solution allows us to suppose that the symmetric solution (10) could be the exact solution of the model. For this reason, we have to study it in detail in order to get more evidence to support this hypothesis.

The extremization with respect to q, x, and m (this last being inside the average and, therefore, for any different ω) leads to a system of self-consistent equations:

$$m_{\omega}=\tanh(\beta\sqrt{q}\omega+\beta x m_{\omega}),$$
$$q=\langle m_{\omega}^2\rangle, \qquad (12)$$
$$x=\frac{1}{\sqrt{q}}\langle\omega m_{\omega}\rangle.$$

This system of equations is solved by q^*, x^*, and m_{ω}^* and the free energy may be written as

$$f=-x^*q^*-\frac{1}{\beta}\langle\Phi(m_{\omega}^*)\rangle.$$

Let us stress that q^* corresponds to a maximum with respect to q while the limit $n\to 0$ has transformed x^* in a minimum with respect to x. Notice that Eqs. (12) have been found out by De Dominics *et al.* [9] (see also [10]) as a first step of a dynamical solution of the ordinary SK model.

For a given ω, the first of Eqs. (12), which refer to the m_{ω}, could have a single solution (a maximum) or three different solutions, depending on the temperature. At low temperature, we have a single solution for $\omega\gtrsim x/\sqrt{q}$ and three solutions for $\omega\lesssim x/\sqrt{q}$. Two of these correspond to a maximum and the third to a minimum, and this introduces an element of uncertainty. We follow the rule of taking the so-

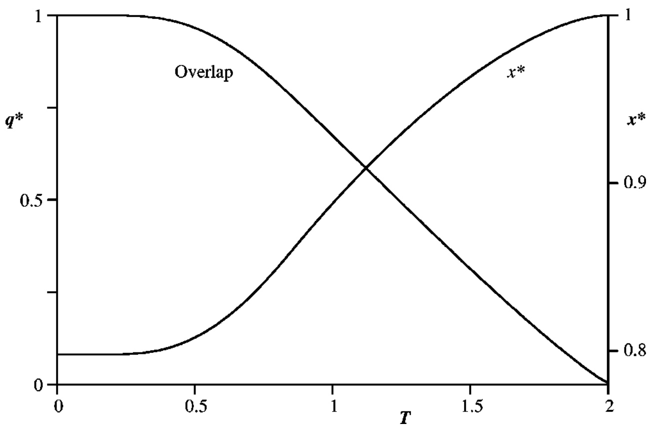

FIG. 1. Overlap q^* and parameter x^* as functions of temperature T for the symmetric solution. The critical temperature below which we have a spin-glass phase ($q^*>0$) turns out to be $T_c=2$.

lution m_ω^* of the first equation, which corresponds to the larger of the two maxima for every given ω.

In Fig. 1, we plot the overlap q^* and the parameter x^* as functions of the temperature $T\equiv 1/\beta$. The spin-glass transition occurs at the critical temperature $T_c=2$, which is the same as that of the SK model.

In Fig. 2, the free energy f and the entropy $S=\langle\Phi(m_\omega^*)\rangle$ are plotted as functions of the temperature T. At $T=0$, the free energy is $f_0=\sqrt{2/\pi}\simeq 0.798$, which is very close to the value of the SK symmetric solution. On the contrary, the entropy simply vanishes at $T=0$ at variance with the SK case, where the negative entropy proves the unphysical nature of the solution in that case.

Let us stress that taking the right extreme point with respect to q, x, and m is a crucial step of the solution, and our choice, previously described, could be inappropriate. In fact, with the limit $n\to 0$, the maximum with respect to x has become a minimum, and this could also happen for some of the m_ω. In this case, one should look for the minimum with respect to the m_ω (or for the second maximum), at least for a subset of ω. Indeed, at this stage, we are not able to give a definite answer regarding this point, which should be an argument for future thorough investigations.

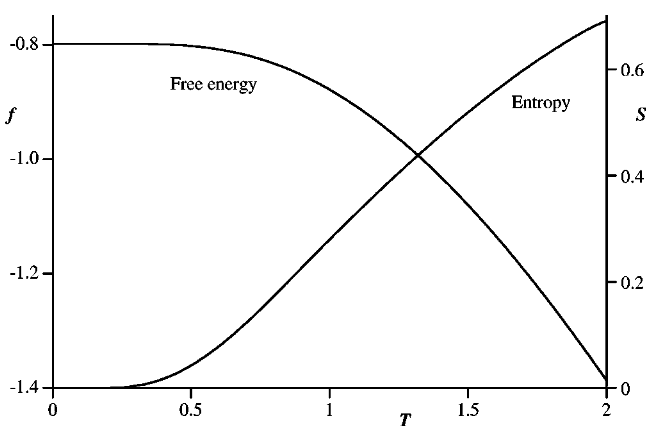

FIG. 2. Free energy f and entropy S as functions of temperature T for the symmetric solution. In the limit $T\to 0$, the solution keeps a physical meaning since the entropy never becomes negative.

VII. CONCLUSIONS

A dynamical approach to our spin-glass model could be of some help in deciding the correct solution. Following Eq. (5), the deterministic dynamics of L magnetizations is

$$m_i(t+1)=\tanh\left[\frac{\beta}{\sqrt{L}}\sum_j J_{i,j}m_j(t)\right],\quad 1\leqslant i\leqslant L.$$

Let us remember that the matrix J has vanishing diagonal entries $J_{i,i}=0$, so that at each step the new value of the individual magnetization m_i does not depend on its previous one. The above dynamics takes advantage of peculiar features. For instance, at each updating it moves m_i in a value corresponding to a minimum free energy with respect to m_i itself, keeping the other magnetizations fixed. Moreover, the free energy always decreases at each updating of a single magnetization.

The dynamics causes the system to evolve toward a fixed point, which is a relative minimum of the free energy (not, in general, a global minimum). Repeating many times this evolution, starting from different initial values for the magnetizations, allows us to find the global minimum corresponding to the solution of the static spin-glass model. Preliminary results seem to suggest that the theoretical symmetric solution of Sec. VI is slightly different from the dynamic solution only for very low temperatures. This does not necessarily imply that the symmetric solution is not the correct one. In fact, in order to avoid finite-size effects, one has to deal with large lattices (large L) in numerical simulations, so that the basin of attraction of the global minimum tends reasonably to become so small that one never uses correct initial conditions in spite of the large number of attempts.

The above-mentioned features make such a dynamics for magnetization versatile and very fast from a numerical point of view. Furthermore, not only is it useful for understanding the associated static model, but it is also interesting in itself. In fact, it describes a dynamical system that monotonically relaxes towards a stable point corresponding to a local minimum of the free energy.

For this reason, it is the ideal candidate for modeling some complex systems, such as natural ecosystems, where each agent or species tries to maximize its own fitness in a given context of other active agents. The fitness corresponds to the individual free energy with changed sign (the part of the free energy that depends on a given magnetization m_i), and the magnetization m_i corresponds to the species degree of specialization. The individual agent or species attempts to improve its own condition, and it happens to push the whole system to maximize the total fitness. This is the very peculiar feature of many real systems that is reproduced by our dynamical model, which also exhibits other realistic peculiarities, such as the fact that the phase space is a landscape of a large number of local maxima of the fitness at low temperature. In the case of a catastrophe (even a small change of the couplings), the system is no longer in a state of maximal fitness, and the evolution restarts towards a different local maximum (a new period of stability in the evolution), which is not necessarily higher than the previous one.

In conclusion, this model seems to be very versatile, since its dynamics could become both a powerful benchmark with which to test general hypotheses about spin glasses, and a paradigmatic model for evolving complex systems.

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